NS – 310

V Semester B.A./B.Sc. Examination, November/December 2016
(Semester Scheme)
(Repeaters – Prior to 2016-17)
(NS 2013-14 and Onwards)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) In a vector space V(F), if $C_1\alpha = C_2\alpha$ and $\alpha \neq 0$ then show that $C_1 = C_2$ where $\alpha \in v$ and $C_1, C_2 \in F$.

2) Show that the subset $W = \{(x_1, x_2, x_3)/x_1 + x_2 + x_3 = 0\}$ of the vector space $V_3(R)$ is a sub-space of $V_3(R)$.

3) Show that (1, 0, 1), (0, 2, 2) and (3, 7, 1) are linearly independent in $V_3(R)$.

4) Find the matrix of the linear transformation $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (x+y, x, 3x - y)$.

5) Determine whether the set {(1, 2, 1), (3, 4, –7), (3, 1, 5)} is a basis of $V_3(R)$.

6) Define rank and nullity of linear transformation $T : U \to V$.

7) Find the unit tangent 't' for the space curve $x = t, y = t^2$ and $z = \frac{2}{3}t^3$.

8) Find the normal and tangent plane to the cylinder $x^2 + y^2 = 4$ at the point $(1, \sqrt{3}, 2)$.

9) If $\vec{r}(t) = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, show that $\vec{r} \times \frac{d\vec{r}}{dt} = \omega(\vec{a} \times \vec{b})$.

10) Find the equation of the tangent plane to the surface $\vec{r} = (u+v)\hat{i} + (u-v)\hat{j} + 4u^2\hat{k}$ at u = 1, v = 2.

11) Find the maximal directional derivative of $x^2y + yz^2 - xz^3$ at (–1, 2, 1).

12) Prove that $\text{div}(\text{curl}\,\vec{F}) = 0$.

13) Find $\nabla^2\phi$ where $\phi = 2x^3y^2z^4$.

14) Show that $\vec{F} = (\sin y + z)\hat{i} + (x\cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational.

15) If $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find curl $(\vec{F})$.

16) If $\phi(x, y, z) = x^2 + \sin y + z$, find grad ϕ at $\left(0, \frac{\pi}{2}, 1\right)$.

17) Define :

i) Complex Fourier transform

ii) Inverse complex Fourier transform of f(x).

18) Find the Fourier transform of $f(x) = e^{-|x|}$.

19) Find the Fourier cosine transform of $f(x) = e^{-ax}$ $(a > 0)$.

20) Find the sine transform of $2e^{-5x} + 5e^{-2x}$.

II. Answer **any four** of the following : (4×5=20)

1) Prove that the inter section of any two sub-spaces of a vector space over a field $\vec{F}$ is a sub-space of the space. Does this property hold for union of two sub-spaces ?

2) Express the vector (3, 5, 2) as a linear combination of the vectors (1, 1, 0), (2, 3, 0), (0, 0, 1) of $V_3(R)$.

3) Find the linear transformation $T : R^2 \to R^2$ such that T(1, 1) = (0, 1) and T(–1, 1) = (3, 2).

4) If $T : V_3(R) \to V_2(R)$ defined by $T(e_1) = (2, 1)$, $T(e_2) = (0, 1)$, $T(e_3) = (1, 1)$. Find the range space, kernel, rank and nullity and verify rank + nullity = dim (domain).

5) Show that $T : V_2(R) \to V_2(R)$ defined by T(x, y) = (3x + 2y, 3x – 4y) is a linear transformation.

6) Find the linear transformation T whose matrix w.r.t. the bases

$B_1 = \{(1, 2, 3), (1, 1, 1), (1, 0, 0)\}$ and $B_2 = \{(1, 1), (1, -1)\}$ is $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}$.

III. Answer **any four** of the following : (5×4=20)

1) State and prove Serret-Frenet formulae for a space curve.

2) For the curve x = a cost, y = a sint, z = bt show that $K = \frac{a}{a^2+b^2}$ and $\tau = \frac{b}{a^2+b^2}$.

3) Find the equation of the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point (1, –1, 2).

4) Find the unit tangent vector at a point t = 2 on the curve $\vec{r}(t) = (t^2+2)\hat{i} + (4t-5)\hat{j} + (2t^2-6t)\hat{k}$.

5) Show that the surface $5x^2 - 2yz = 9x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at (1, –1, 2).

6) Express the vector $\vec{f} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in terms of cylindrical co-ordinates.

IV. Answer **any three** of the following : (5×3=15)

1) Find the directional derivative of the function $\phi = xy^2 + yz^3$ at (2, –1, 1) in the direction of $2\hat{i} + \hat{j} + 2\hat{k}$.

2) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

3) Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

4) Prove that $\text{div}(\vec{f} \times \vec{g}) = \vec{g}.\text{curl}\,\vec{f} - \vec{f}.\text{curl}\,\vec{g}$.

5) Derive the expression for divergence of a vector function in orthogonal Curvilinear co-ordinates.

V. Answer **any three** of the following : **(3x5=15)**

1) Express $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate

$\int_0^\infty \frac{\sin x}{x} dx$.

2) Find the complex Fourier transform of $f(x) = e^{-a^2x^2}$, where 'a' is a positive constant.

3) Find the Fourier cosine transform of e^{-x^2}.

4) Find the Fourier sine transform of $e^{-|x|}$ hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$; $m > 0$.

5) Prove that $F_C[f''(x)] = -\sqrt{\frac{2}{\pi}} f'(0) - \alpha^2 F_C[f(x)]$.

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